

EVAPORATIVE COOLING OF GASES

G. A. Mikhailovskii

Inzhenerno-Fizicheskii Zhurnal, Vol. 11, No. 1, pp. 22-29, 1966

UDC 533.6

Expressions are given for the adiabatic exponent in equations with various independent parameters relating to processes involving evaporative cooling of gases. Some peculiarities of such processes and unusual effects associated with them are discussed.

In evaporative cooling of a gas in a high-velocity stream, or in the compression process in a compressor stage, or, finally, simply in a pipeline through which a gas is transferred from one part of a plant to another, it is necessary to take into account the peculiarities of the process, and, in particular, the thermodynamic cooling effect. This last phenomenon is sometimes the main objective of water injection, an example being the aerothermopressor.

The thermogasdynamic cooling effect has not been studied very closely. An investigation of adiabatic processes with phase transitions sheds some light on its complex laws and is the first stage in an analytical treatment.

The processes in evaporative cooling of gases are adiabatic. Because of the absorption of the greater part of the heat by the evaporating liquid, the caloric properties of the mixture of gas and vapor vary over a wide range.

These peculiarities of the mixture are described by the value of the adiabatic exponent. The latter, as is shown by investigation, may take values of both 0 and $\pm\infty$, and all positive and all negative values, depending on the intensity of evaporation. Certain finite positive values of the adiabatic exponent are attained when the intensity of evaporation with respect to temperature $(\partial d_e/\partial T)_{dq=0} \rightarrow \pm\infty$. In equations of the adiabatic curve with various independent parameters, the adiabatic exponent assumes various values.

For a mixture containing 1 kg of dry gas, we will write the following equations:

$$dQ = C_u dT + pdV = 0, \text{ kJ/kg of dry gas, (1)}$$

$$dQ = C_i dT - Vdp = 0, \text{ kJ/kg of dry gas, (2)}$$

$$pV = RT, \text{ (3)}$$

$$C_i - C_u = R + p_e v_e \xi, \text{ kJ/kg of dry gas} \cdot \text{degree (4)*}$$

where

$$C_u = \left(\frac{\partial U}{\partial T} \right)_{dq=0} = C_v + u_e \xi, \text{ (5)}$$

$$C_i = \left(\frac{\partial I}{\partial T} \right)_{dq=0} = C_p + i_e \xi, \text{ (6)}$$

$R = R_g + d_e R_e$ is the gas constant, depending in a given case on the vapor content, kJ/kg of dry gas.

From (3) we obtain

$$dT = (pdV + Vdp - TdR)/R. \text{ (7)}$$

With the aid of (1)-(7) we find the equations of an adiabatic process with evaporative cooling, and expressions for the adiabatic exponent. We accomplish the derivations by three methods having different original equations. In each of the three cases we find three equations of the process, corresponding to three possible combinations of the variable parameters.

1) The original equation is (1). Substituting the value of dT from (7), we find

$$(C_u + R) \frac{dV}{V} + C_u \frac{dp}{p} - C_u \frac{dR}{R} = 0.$$

Using (4), we obtain

$$k_2 \frac{dV}{V} + \frac{dp}{p} - \frac{dR}{R} = 0, \text{ (a)}$$

where

$$k_2 = (C_i - p_e v_e \xi)/C_u \text{ (8)}$$

is the local adiabatic exponent. It is convenient to designate it by the subscript 2. (Other expressions will be obtained later, which we shall designate k_1 and k_3 .)

If the quantity k_2 may be assumed constant during the process or in a certain part of it, then from (a), after integration, we obtain the adiabatic curve equation with independent parameters p and V:

$$pV^{k_2}/R = \text{const.}$$

From this expression, with the aid of (3), we may obtain equations with variables T, V and T, p:

$$TV^{k_2-1} = \text{const.}$$

$$T(R/p)^{(k_2-1)/k_2} = \text{const. (9)}$$

The adiabatic exponent k_2 in these equations must be calculated from (8). We note that of the three equations obtained for the process, only one, with independent variables T and V, has the usual form. The other two equations are complicated by the fact that they contain a third variable quantity R.

2) The original equation is (2). Substituting the temperature differential from (7), we obtain

$$(C_i - R) \frac{dp}{p} + C_i \frac{dV}{V} - C_i \frac{dR}{R} = 0$$

*See [1] with regard to (4)-(6).

Table 1

Adiabatic Exponents

Independent variables	General expression	For special adiabatic processes				$\varphi = \text{const}$ k_p
		$V = \text{const}$ k_V	$p = \text{const}$ k_p	$T = \text{const}$ k_T	$d_n = \text{const}$ k_d	
p, V	$k_{1V} = \frac{C_i}{C_u}$	∞	0	$\frac{i e}{u e}$	$\frac{C_p}{C_V}$	$\frac{k_d(1+a) - \frac{i e}{u e}(1-b)}{a-b}$
T, V	$k_{2V} = \frac{C_i - p_e v_e \xi}{C_u}$	∞	$\frac{p_e v_e \xi}{C_u}$	1	$\frac{C_p}{C_V}$	$\frac{k_d(1+a) - (1+b)}{a-b}$
T, p	$k_{3p} = \frac{C_i}{C_u + p_e v_e \xi}$	$\frac{C_i}{p_e v_e \xi}$	0	1	$\frac{C_p}{C_V}$	$\frac{k_d(1+a) - (1+b)}{a - \frac{i e}{u e} b}$

Remark: $a = p_e v_e / u_e$; $b = p d_e (\partial v_e / \partial T) \varphi / C_V$. The exponents k_{2p} and k_{3V} take negative values. A process with $\varphi = \text{const}$ is possible at temperatures not in excess of the critical temperature for the vapor.

and, expressing the difference term in brackets in terms of (4), we find

$$k_3 \left(\frac{dV}{V} - \frac{dR}{R} \right) + \frac{dp}{p} = 0, \tag{b}$$

where

$$k_3 = C_i / (C_u + p_e v_e \xi) \tag{10}$$

is also the local adiabatic exponent.

If k_3 may be assumed to be constant, then, integrating (8), we obtain an equation with variables p and V

$$p(V/R)^{k_3} = \text{const.}$$

Similarly, we also find the following two equations:

$$\begin{aligned} T/p^{(k_2-1)/k_3} &= \text{const,} \\ T(V/R)^{k_3-1} &= \text{const,} \end{aligned} \tag{11}$$

where the value of the adiabatic exponent k_3 must be determined from (10).

In this case, of the three equations obtained, that with independent variables T and p has the usual form. However, the adiabatic exponent is now expressed by another relation and therefore differs numerically from k_2 .

3) The original equations are those of an adiabatic curve of the usual type, (9) and (11), with variables

T, V and T, p . These equations may be written in the following form:

$$TV^{R/C_u} = \text{const}; \quad Tp^{-R/C_i} = \text{const.}$$

If the first one is divided by the second and then a root of degree R/C_i is extracted from both sides of the equality obtained, we obtain

$$pV^{k_1} = \text{const}, \tag{12}$$

where

$$k_1 = C_i/C_u. \tag{13}$$

Making a substitution in (12), in one case of the volume and in the other of the pressure from (3), we obtain equations with variables T, V and T, p in the following form:

$$\begin{aligned} RTV^{k_1-1} &= \text{const,} \\ RT/p^{(k_1-1)/k_1} &= \text{const.} \end{aligned}$$

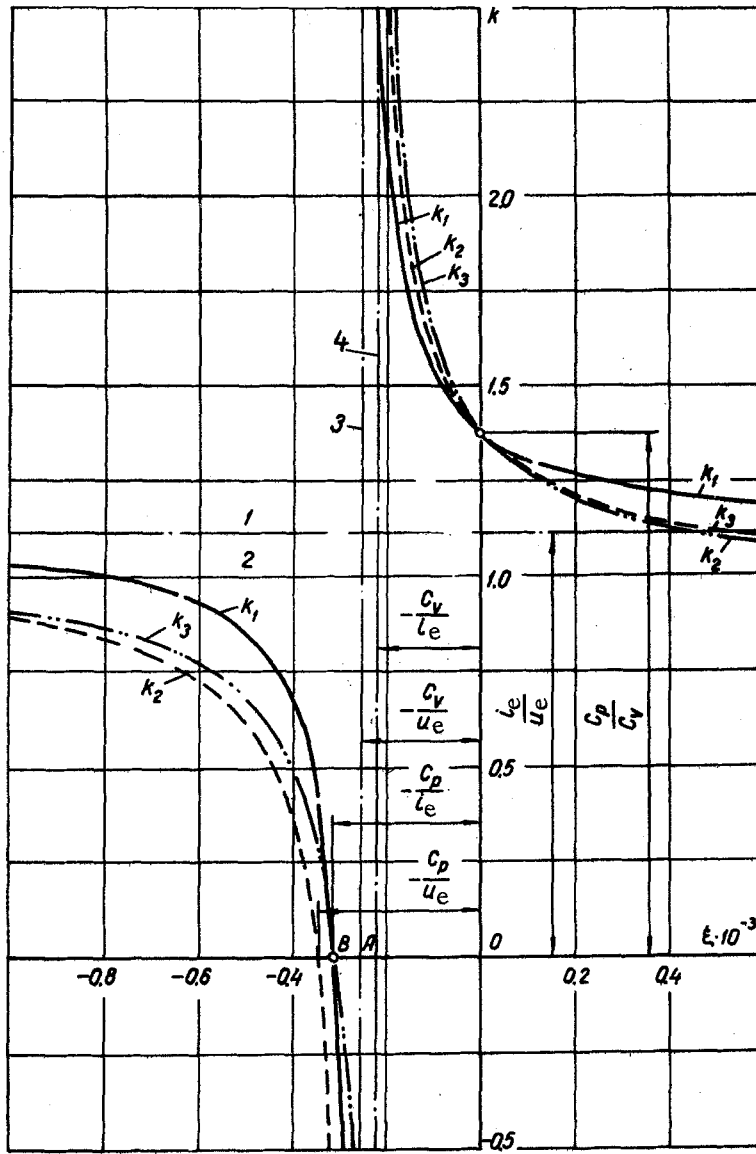
Consequently, in this case also we obtain only one equation of the usual type for the adiabatic curve—the equation with variables p and V (12). We again obtain a new expression for the adiabatic exponent.

Thus, it follows from all that has been said, that if we take the ordinary form of the equation with variables T, V ; T, p ; and p, V , the adiabatic exponent in these three cases will be different. The corresponding expressions are shown in Table 1.

Table 2

Coordinates of Center of Hyperbola

Coordinates of center	For $k_1 = j_1$ (ξ)	For $k_2 = j_2$ (ξ)	For $k_3 = j_3$ (ξ)
ξ_c	$\frac{C_V}{u_e}$	$\frac{C_V}{u_e}$	$\frac{C_V}{i e}$
k_c	$\frac{i e}{u_e}$	1	1



Values of local adiabatic exponents for processes with evaporation of injected water (1, 2, 3, 4— asymptotes respectively for k_1 , for k_2 and k_3 , for k_1 and k_2 , and for k_3).

Table 3

Limits of Adiabatic Exponent Values in Processes with Evaporation

Process group	ξ	k_1	k_2	k_3
I	$-\infty - - \frac{C_p}{i_e}$	$\frac{i_e}{u_e} - 0$	$1 - k_{2p}$	$1 - 0$
II	$\frac{C_p}{i_e} - \frac{C_V}{u_e}$	$0 - - \infty$	$k_{2p} - - \infty$	$0 - k_{3V}$
III	$-\frac{C_V}{u_e} - 0$	$+ \infty - k_d$	$+ \infty - k_d$	$k_{3V} - k_d$
IV	$0 - + \infty$	$k_d - \frac{i_e}{u_e}$	$k_d - 1$	$k_d - 1$

Remark: k_d , k_{2p} and k_{3V} are from Table 1.

Table 1 also gives the relations for the adiabatic exponent in particular cases of an adiabatic process. These relations are easily obtained from the general expressions, bearing in mind [1] that in an adiabatic-isochoric process $C_u = 0$, in an adiabatic-isobaric process $C_i = 0$, in an adiabatic-isothermal process $\xi = \infty$, and in an adiabatic process at constant vapor content $\xi = 0$. Somewhat more complex relations are obtained for the adiabatic exponent $k\varphi$ in a process at constant relative humidity φ . We will not give the method of obtaining these here, however.

When there is variation in the intensity of phase transition ξ , the temperature coefficients C_u and C_i vary considerably. This has an effect on the adiabatic exponent, as follows from the general expressions for k_1 , k_2 , and k_3 . This dependence is shown graphically in the figure.

A calculation was made for air under the following conditions: $p = 2.25 \cdot 10^5 \text{ N/m}^2$, $t = 500^\circ \text{ C}$, $d_e = 0.02 \text{ kg/kg}$ of dry air, and for various values of the intensity of evaporation $\xi = (-1 + 0.6) \cdot 10^{-3} \text{ kg/kg}$ of dry air per degree.

The relation $k = f(\xi)$ in all three cases is expressed in terms of linear-fractional functions, graphs of which are equilateral hyperbolas with asymptotes parallel to the coordinate axes. To find the asymptotes of these hyperbolas, it is sufficient to find the coordinates of their centers in each of the three cases. Since we know the equations of the hyperbolas (Eqs. (8), (10), and (13)), this is easily accomplished by ordinary mathematical methods. The center coordinates found for each hyperbola are shown in Table 2.

From Table 2 and the figure it may be seen that the abscissa of the center of the two hyperbolas relating to k_1 and k_2 is $\xi = -C_V/u_e$. But from [1] it is known that for such a phase transition rate, an adiabatic-isochoric process occurs. Therefore k_1 and k_2 pass through $\pm\infty$ in an adiabatic-isochoric process. The point A on the abscissa axis corresponds to this process.

It is known that the region of the graph between the asymptote passing through the point A and the coordinate axis (with $\xi = -C_V/u_e - 0$) is associated with expansion processes. Therefore the sections of the curves lying in this region determine the value of the adia-

batic exponent in expansion processes, and those to the right and left of this region—in compression processes.

The curves in the figure intersect the abscissa axis to the left of point A, and therefore, in processes with evaporation of an injected liquid, the adiabatic exponents k_1 , k_2 and k_3 may take zero values only in compression processes (at reduced temperature, as will be shown below).

The exponents k_1 and k_2 vanish at a single value of ξ , and in this case the corresponding curves intersect (point B in the figure). To find the value of ξ corresponding to this point, we equate to zero the expressions for k_1 or k_3 from (10) or (13). We find as a result that $C_i = 0$, and expressing C_i according to (6), we obtain

$$\xi = -C_p/i_e.$$

The values obtained for C_i and ξ define an adiabatic-isobaric process [1], to which the point B on the abscissa axis corresponds.

The adiabatic exponent k_2 takes a zero value, as follows from (8) and (6), when

$$C_i - \rho_e v_e \xi = C_p + u_e \xi = 0$$

or for an evaporation rate of

$$\xi = -C_p/u_e.$$

All three curves intersect on the ordinate axis. Since then $\xi = 0$, $k = C_p/C_V$, i. e., it takes on the usual expression for the adiabatic exponent in a process without phase transition.

A graphical relation is shown in the figure for a gas temperature (500° C) above the critical temperature of water vapor (374° C), and therefore there is no physical state characterized by the relative humidity. In this case adiabatic processes with evaporative cooling must be divided into four groups. The ranges of adiabatic exponent values for each of the four groups are given in Table 3.

Only group III contains expansion processes. Compression processes of the first two groups differ from the compression processes of group IV as regards the

sign of the temperature increment. Processes of groups I and II take place with $dT < 0$, due to the considerable cooling. A convenient cooling rate criterion is the volume phase transition rate

$$\omega = \left(\frac{\partial d_e}{\partial V} \right)_{dq=0}$$

Processes of the first two groups occur at a cooling rate

$$|\omega| > p/u_e$$

In the processes of group IV, $dT > 0$, since the cooling rate is relatively low:

$$|\omega| < p/u_e$$

The boundary between processes of groups I and IV (beyond the limits of the graph) is an isothermal process in which the left and right sides of the given inequalities are equal. This is obtained with $\xi = \pm\infty$.

With these parameters of state, for which the curves in the figure were constructed, the numerical values of the quantities of Table 3 are as follows: $\xi_p = -C_p/i_e = -0.31 \cdot 10^{-3}$ kg/kg of dry air · degree = -0.31 g/kg of dry air · degree; $\xi_V = -C_V/u_e = -0.25 \cdot 10^{-3}$ kg/kg of dry air · degree = -0.25 g/kg of dry air · degree; $k_{1T} = i_e/u_e = 1.116$; $k_{2p} = -0.602$; $k_d = 1.38$; $k_{3V} = -2.29$; the volume phase transition rate $\omega = -0.722$ kg/m³.

Table 3 makes a special distinction for processes of group II for which all three adiabatic exponents take negative values. The quantities k_2 in group I and k_3 in group III take negative values in part.

Thus, we come to the following conclusions:

- 1) The local adiabatic exponent takes values from $-\infty$ to $+\infty$, depending on the evaporation rate.
- 2) In the adiabatic curve equations with different independent variables ($p, V; T, V; p, T$), the adiabatic exponent takes different numerical values.

- 3) In group II processes, i. e., with an evaporation rate

$$-\frac{C_p}{i_e} < \left(\frac{\partial d_e}{\partial T} \right)_{dq=0} < -\frac{C_V}{u_e}$$

all three adiabatic exponents take negative values.

- 4) These unusual properties associated with processes with evaporative cooling lead to unexpected flow effects. Thus, for example, the pressure may drop during compression, and the reduction in pressure may be accompanied, not by an increase, as is usual, but by a decrease in stream velocity, and so forth.

NOTATION

C_u and C_i are, respectively, temperature coefficients of internal energy and enthalpy, kJ/kg of dry gas · degree; C_V and C_p are the isochoric and isobaric heat capacities, relative to 1 kg of dry gas, in phase transition processes, kJ/kg of dry gas · degree; $\xi = (\partial d_e / \partial T)_{dq=0}$ is the phase transition rate; d_e is the vapor content in kg/kg of dry gas; p_e and v_e are the partial pressure, N/m², and specific volume, m³/kg in mixture conditions; u_e and i_e are the internal energy and enthalpy of vapor, kJ/kg; φ is the relative humidity of gas.

REFERENCES

1. G. A. Mikhailovskii, Thermodynamic Calculations for Processes in Vapor-Gas Mixtures [in Russian], Mashgiz, 1962.
2. L. A. Vulis, Thermodynamics of Gas Streams [in Russian], Energoizdat, 1950.
3. G. A. Mikhailovskii, Intern. Journ. of Heat and Mass Transfer, 7, 1964.

28 June 1965

Institute of Water Transport,
Leningrad